

On a general way of two-particle problem reduction

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Abstract

The problem of the description of two interacting particles is considered. It is shown that it can be reduced to the description of one particle in an external static potential even in a relativistic case. The method is based on the Maupertuis least action principle. The proposed method is verified for known problems in two-particle problems in classical mechanics, as well as in non-relativistic and relativistic quantum mechanics.

1 Introduction

The problem of two-particle mutual interactions in physics is very old. The traditional way to solve it is to consider it in the *center-of-mass system* (c.m.s.) of reference [1]. After a simple substitution of variables and excluding the motion of the system as a whole, one obtains the equation for an effective particle with a *reduced mass* in an external potential of a fixed center. It is well known how to do that in the classical mechanics (in the two-body Kepler problem, for example) and in the non-relativistic quantum mechanics in the two-particle Schrödinger equation. But in a relativistic case in the c.m.s. we do not have so clear simplification of our problem. Even to write down a relativistic Schrödinger-like two-particle equation is a serious problem. Of course there are many approaches to the relativistic two-particle problem: quasi-potential approach [2] suggested by Logunov and Tavkhelidze, the Bethe-Salpeter equation [3], the Droz-Vincent-Komar-Todorov equation [4, 5, 6] for scalar particles, the two-body Dirac equations [7, 8] for spinor particles and many other.

Below we propose a simple method which allows to reduce any two particle problem to the description of an (effective) particle in a static central potential. The idea is to exploit the duality between the scattering channel and the bound state being the two branches of the same two-particle problem solution. The method works in the same way as in classical and quantum mechanics, as well as in the quantum field theory. Our restrictions are natural: the particles should be point-like (or can be effectively considered as point-like) and their interaction should be local.

The paper is organized as follows. In the second section of this paper we will consider the general two-particle problem from the point of view of the least action principle. From a class

of possible ways of the elimination of the degrees of freedom, corresponding to the motion of the two-particle system as a whole, we will choose one. As an illustration the classical Kepler problem will be considered. In the third section we will imply our method to the following problems: the two-particle Schrödinger equation with the Coulomb potential involved; the problem of electromagnetic interactions between a spinor and a scalar relativistic charged particles; and than — the same problem for two spinor particles. The last section of the paper contains conclusions and a discussion.

2 The two-particle problem

Hamilton's theorem proofs that the *two-body central force problem* in classical mechanics can be reduced to the equivalent one-body problem, see e.g. Ref. [9]. The statement can be easily generalized for non-relativistic quantum mechanics of two particles. But the reduction of a relativistic two-particle problem still causes the appearance of new approaches.

Let us suppose that we have a certain Lagrangian for our two particles which includes an interaction. Usually one constructs it in an inertial reference frame (RF) in order to avoid additional "non-inertial" contributions. For an illustration we take now the classical Kepler problem. The Lagrangian can be written as

$$\mathcal{L} = \frac{m_1 \dot{\vec{r}}_1^2}{2} + \frac{m_2 \dot{\vec{r}}_2^2}{2} - U, \quad U = -\frac{m_1 m_2}{r}, \quad (1)$$

where m_1 and m_2 are the masses of our particles, \vec{r}_1 and \vec{r}_2 are their coordinates, r is the absolute value of the relative coordinate ($r \equiv |\vec{r}_2 - \vec{r}_1|$), and the dots over \vec{r}_1 and \vec{r}_2 denote time derivatives. We chose unity for the value of the coupling constant.

Then we can use the least action principle and write down the equation for the action variation. We will use here the fact of the energy conservation in our system and look for a static solution. So, we can use the original action definition due to Maupertuis, Euler and Lagrange instead of the Hamilton's one. The so called *abbreviated action* of a conservative system does not contain the time variable. Using the variational principle for it, one can obtain particle trajectories. The time dependence could be restored after all. Note, that in the quantum mechanics in a stationary case, one also often looks for observable quantities and forget about the time dependence (in the Schrödinger representation).

The abbreviated action for the Kepler problem reads

$$\begin{aligned} \mathcal{S} &= \int \sqrt{2(E - U)[m_1(d\vec{r}_1)^2 + m_2(d\vec{r}_2)^2]} \\ (d\vec{r}_i)^2 &\equiv (dx_i)^2 + (dy_i)^2 + (dz_i)^2, \end{aligned} \quad (2)$$

where the integral has to be taken between two points on a trajectory. E is the energy of the two-particle system.

Our approach is based on the following statement: the problem of the action variation can be considered in *any reference frame* even in a non-inertial one. That was shown by Lagrange and follows just from the fact that the action is a scalar. In other words, we have written the action in a certain RF and after that we can take its variation in any reference frame. And

there is only one reason to make a concrete choice: one should choose the RF being suitable for the given problem.

Following the tradition one can substitute the variables in Eq. (2) as

$$\vec{r}_1, \vec{r}_2, \vec{p}_1, \vec{p}_2 \longrightarrow \vec{R}, \vec{r}, \vec{P}, \vec{p}, \quad (3)$$

where

$$\begin{aligned} \vec{R} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}, & \vec{r} &= \vec{r}_2 - \vec{r}_1, \\ \vec{P} &= \vec{p}_1 + \vec{p}_2, & \vec{p} &= \frac{m_1 \vec{p}_2 - m_2 \vec{p}_1}{m_1 + m_2}. \end{aligned}$$

The substitution for the particle momenta \vec{p}_1 and \vec{p}_2 is shown for a completeness. Further we choose the c.m.s. from all possible reference frames and obtain the action in the form

$$\mathcal{S} = \int \sqrt{2(E - U)} \mu \, d\vec{r}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}. \quad (4)$$

This expression has the form of a one-particle abbreviated action. The subsequent procedure is straight forward, we are not going to repeat it in details. It is well known that the c.m.s. is rather suitable for problems which have the same kinetic part in their Lagrangians (for the two-particle Schrödinger equation, for instance). But in a relativistic case we have another structure of the kinetic part. And the choice of the c.m.s. as a reference frame does not reduce a relativistic two-particle problem to a simple one-particle problem¹.

Let us consider another possible way to choose the RF. The class of reference frames, where

$$A\vec{p}_1 + B\vec{p}_2 = 0, \quad (5)$$

has a simple physical interpretation. For example, choosing $A = 1$ and $B = 0$ one obtains the rest frame of the first particle, choosing $A = B = 1$ one obtains the c.m.s. The corresponding substitution for coordinates follows from the conditions

$$\vec{p}_1 \vec{r}_1 + \vec{p}_2 \vec{r}_2 = \vec{P} \vec{R} + \vec{p} \vec{r} \quad \text{and} \quad \vec{r} = \vec{r}_2 - \vec{r}_1. \quad (6)$$

It is easy to show that all such choices of reference frames are equivalent in a non-relativistic case: they all reduce the problem to a one-particle one (the difference would be only in different expressions for *reduced masses*), and the subsequent turn back to an inertial (c.m.s.) system of reference is rather simple, it will be discussed below. We will do now a concrete choice. We choose as the working reference frame the rest frame of one of the particles, of the first one to be definite. There the two-particle action presented in eq. (2) takes the form

$$\mathcal{S} = \int \sqrt{2(E - U)m_2} \, d\vec{r}_2, \quad (7)$$

which is just the form of the action for the motion of the second particle in the potential created by the first one. It is a certain degenerated case, the reduced mass now coincides

¹Certainly, it reduces the number of the degrees of freedom but it does not lead to the one-particle Dirac equation, for instance.

with the mass of the second particle. One can see that the expression does not depend on the relation between the masses. We would obtain the same action as for the case $m_1 \gg m_2$ as well as for the case $m_1 \ll m_2$. Solving the Euler–Lagrange equations corresponding to the action we obtain a trajectory (a conic curve) which depends actually on three parameters: the energy E_2 , the angular momentum M_2 and the mass m_2 . In the polar coordinates we have

$$\frac{M_2^2}{m_2 |\vec{r}_2|} = 1 + \sqrt{1 + \frac{2E_2 M_2^2}{m_2}} \cos \varphi. \quad (8)$$

We have to underline that the trajectory found using eq. (7) deals nothing with the *real* trajectory of the second particle in the rest RF of the first one, it expresses the mathematical solution of the problem, defined by Lagrangian (1) and acquires a physical meaning only after the proper transformation into an inertial RF.

So, we have to go back to the c.m.s. We can note that formally we should substitute the mass m_2 by the reduced mass μ in the solution. But we can consider this problem from another side. The solution, that we obtained, works as for *bound states* as well as for *scattering states*. Let us look at the final scattering state. Here we have two free particles and, so, it is simple to go from the rest RF of the first particle into the c.m.s. or into any RF defined by relation (5) using simple rules. Here our non-inertial (in general) reference frame appears to be an inertial one. Let us consider the relations between the parameters of the found solution in our RF and in the c.m.s. A simple Galilean transformation gives us

$$E_{CM} = E_2 \frac{\mu}{m_2}, \quad M_{CM} = M_2 \frac{\mu}{m_2}, \quad m_{CM} = \frac{M_{CM}^2}{2E_{CM}\rho^2} = m_2 \frac{\mu}{m_2} = \mu, \quad (9)$$

where $E_2 = m_2 v_2^2/2$ and $M_2 = m v_2 \rho$ are the energy and the angular momentum of the second particle in the rest RF of the first one (in the asymptotic state) (they are equal to the corresponding parameters of the whole system in the chosen RF). The impact parameter ρ is defined in the asymptotic initial (or final) state, it is the same in both systems. E_{CM} and M_{CM} are the energy and the angular momentum of the two-particle system in the c.m.s. Note that m_2 and μ in both cases are not physical masses but some parameters of the solutions. The energy and the angular momentum are conserved in our problem. And we can use the relations for them as for a scattering case as well as for a bound state, because they *should* obey the same laws being only different cases of the general solution. Here we rely upon analytical properties of our solution.

The substitutions defined by Eq. (9) transform the effective trajectories (8) obtained from variation of action (7) into the well known trajectories of the effective particle with the reduced mass μ in the c.m.s. We can see that the suggested approach works in the classical mechanics and it does not cause any difficulty. We constructed a simple method of the two-particle problem reduction. Now we will consider some examples from quantum mechanics.

3 Two-particle problem in quantum mechanics

3.1 Two-particle Schrödinger equation

Let us take the non-relativistic Schrödinger equation for two charged particles

$$\left(-\vec{\nabla}_1^2 \frac{1}{2m_1} - \vec{\nabla}_2^2 \frac{1}{2m_2} - \frac{\alpha}{r}\right) \Psi(\vec{r}_1, \vec{r}_2) = E \Psi(\vec{r}_1, \vec{r}_2), \quad (10)$$

$$\vec{\nabla}_i^2 \equiv \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2} + \frac{\partial^2}{\partial z_i^2}, \quad \alpha \equiv \frac{e_1 e_2}{4\pi},$$

where the units $c = \hbar = 1$ are chosen; e_1 and e_2 are the absolute values of the particle charges (we assumed the attractive interaction, in the repulsion case one has to change the sign of α).

The equation expresses also the least action principle, it is written just in the Hamilton form. So, as we learned, we can try to solve it in any reference frame. Consider again the rest reference frame of the first particle ($\vec{p}_1 = 0$, $\vec{r}_1 = 0$). There we have

$$\begin{aligned} \Psi(\vec{r}_1, \vec{r}_2) &= \Psi_1(\vec{r}_1) \Psi_2(\vec{r}_2), & \Psi_1(\vec{r}_1) &= \exp\{-i\vec{p}_1 \vec{r}_1\}, \\ \vec{r} &= \vec{r}_2 - \vec{r}_1 = \vec{r}_2, & E &= E_2. \end{aligned} \quad (11)$$

Thus we obtain an equation of the exactly one-particle form

$$\left(-\vec{\nabla}_2^2 \frac{1}{2m_2} - \frac{\alpha}{|\vec{r}_2|}\right) \Psi_2(\vec{r}_2) = E_2 \Psi_2(\vec{r}_2). \quad (12)$$

We should underline once more that this equation describes a non-physical object, it has the same sense as an equation for a particle with a *reduced mass*.

The discrete spectrum corresponding to eq. (12) reads

$$E_2 = -\frac{\alpha^2}{2n^2} m_2. \quad (13)$$

To obtain the observable spectrum we should return into the c.m.s. We refer again to a scattering state, make there the Galilean transformations (9) and obtain just

$$E_{CM} = -\frac{\alpha^2}{2n^2} \mu. \quad (14)$$

The direct relativization of the problem, i.e. the relativistic problem for two charged scalar particles was considered in our approach in Ref. [10]. There we made another choice of a reference frame from the class given by Eq. (5) — we took the RF of *equal velocities* ($A = 1/m_1$, $B = -1/m_2$). The problem was explicitly reduced to the one-particle Klein-Fock-Gordon equation. The comparison with the case of the rest RF choice was presented also. The c.m.s. spectrum of this system received earlier in another approach [6] was reproduced.

3.2 One scalar and one spinor charged particles

Let us consider now the system consisting of a scalar and a spinor charged particles with masses m_1 and m_2 , respectively. Here we can choose either the rest frame of the scalar particle, or the

rest frame of the spinor one. We should obtain coinciding observable quantities after transition to the c.m.s. That will be a nontrivial test of our method.

We are not going to write any relativistic two-particle equation. We are going to solve a relativistic two-particle problem reducing it to a one-particle problem from the beginning. We know from the general quantum mechanics that there *should be* a relativistic Schrödinger-like two-particle equation for our problem. But we know also that we can consider that equation in the rest reference frame of one of the particles. And in the chosen reference frame, we automatically have just the one-particle relativistic equation describing the motion of the second particle in the field of the first one.

Let us take first the rest RF of the scalar particle. There we have the Dirac equation for the spinor particle in the static Coulomb potential of the scalar one:

$$(\vec{\alpha}\vec{p}_2 + e_2 A^0 + \beta m_2)\Psi = E_2 \Psi, \quad A^0 = -\frac{e_1}{4\pi|\vec{r}_2|}. \quad (15)$$

The exact solution of the problem is well known. For the discrete spectrum one has

$$E_2 = m_2 \left[\sqrt{1 + \left(\frac{\alpha}{\gamma + n_r} \right)^2} \right]^{-1}, \quad (16)$$

$$n_r = n - k, \quad k = j + \frac{1}{2}, \quad \gamma = \sqrt{k^2 - \alpha^2},$$

where n and j are the principal and the total-angular-momentum quantum numbers, respectively.

In a relativistic case we can also consider a scattering state as in Eq. (9) and derive the rules of returning to the c.m.s. For wave functions we obtain simple rules being just analogous to the well known transformations between the laboratory system and the center-of-mass one. To control the energy of our system it is convenient to use the Mandelstam invariant variable s ,

$$s = E_{CM}^2 = (p_1 + p_2)^2, \quad (17)$$

where p_1 and p_2 are the four-momenta of our particles, which can be defined in any reference frame. So we obtain the spectrum of our two-particle bound state in the c.m.s.

$$\begin{aligned} E_{CM} &= \sqrt{s} = \left[m_1^2 + m_2^2 + 2m_1 E_2 \right]^{\frac{1}{2}} = m_1 + m_2 \\ &- \frac{\alpha^2}{2n^2} \mu - \frac{\alpha^4}{2n^3} \mu \left(\frac{1}{k} - \frac{3}{4n} + \frac{m_1 m_2}{4n(m_1 + m_2)^2} \right) - \frac{\alpha^6}{2n^3} \mu \left[\frac{1}{4k^3} + \frac{3}{4nk^2} \right. \\ &- \left. \frac{3}{2n^2 k} + \frac{5}{8n^3} + \frac{\mu}{(m_1 + m_2)} \left(\frac{1}{2n^2 k} - \frac{3}{8n^3} \right) + \frac{\mu^2}{(m_1 + m_2)^2} \frac{1}{8n^3} \right] + \dots \end{aligned} \quad (18)$$

Now let us consider our problem in the rest RF of the spinor particle. Here we have the Klein-Fock-Gordon equation for the scalar particle moving in the static potential created by the spinor one. We have now a small additional *magnetic* term in the potential due to the spin. The equation reads

$$\begin{aligned} [(p_1^\mu - e_1 A^\mu)^2 - m_1^2] \Psi &= 0, \\ A^0 &= -\frac{e_2}{4\pi r_1}, \quad \vec{A} = -\frac{[\vec{\mu} \vec{r}_1]}{4\pi r_1^3} = -\frac{e_2}{2m_2} \cdot \frac{[\vec{s} \vec{r}_1]}{4\pi r_1^3}, \end{aligned} \quad (19)$$

where $\vec{\mu}$ is the magnetic moment of the spinor particle, \vec{s} is its spin, $r_1 \equiv |\vec{r}_1|$.

It is known that the solution of the Klein–Fock–Gordon equation with the Coulomb potential (A^0) has a divergence of the wave function in the zero point. But the additional *magnetic* term (\vec{A}) makes the solution to be convergent in our case. We will consider that additional term as a perturbation, it is possible for the calculations of the spectrum. The solution for the spectrum of Eq. (19) if we took only the Coulomb part of the potential would be the following

$$E_1 = m_1 - \frac{\alpha^2}{2n^2}m_1 - \frac{\alpha^4}{2n^3}m_1\left(\frac{2}{2l+1} - \frac{3}{4n}\right) + \dots \quad (20)$$

The transformation into the c.m.s. gives

$$E'_{CM} = m_1 + m_2 - \frac{\alpha^2}{2n^2}\mu - \frac{\alpha^4}{2n^3}\mu\left(\frac{2}{2l+1} - \frac{3}{4n} + \frac{m_1m_2}{4n(m_1+m_2)^2}\right) + \dots, \quad (21)$$

where l is the orbital angular momentum quantum number. The difference between spectra (18) and (21) is due to the contribution of the additional term in the potential which is not accounted in the last formula. Let us calculate it in the perturbation theory taking the solution of the non-relativistic Schrödinger equation as the ground state. So, we have to calculate the average value of the operator

$$\frac{-2e_1(\vec{p}_1\vec{A})}{2m_1} = \frac{\alpha(\vec{p}_1[\vec{\mu}\vec{r}_1])}{m_1r_1^3} = \frac{\alpha}{2m_1m_2r_1^3}(\vec{l}_1\vec{s}) = \frac{\alpha}{2m_1^2r_1^3}(\vec{l}\vec{s}), \quad (22)$$

where we divided by $2m_1$ in order to normalize the Klein–Fock–Gordon equation to the Schrödinger one. We used the relation between angular momenta in different reference frames

$$\vec{l}_1 = \frac{m_2}{m_1}\vec{l}_{CM} \equiv \vec{l}. \quad (23)$$

Using the standard technique we get

$$\begin{aligned} \delta E &= \left\langle \frac{\alpha}{2m_1^2r_1^3}(\vec{l}\vec{s}) \right\rangle = \frac{\alpha}{2m_1^2} \langle (\vec{l}\vec{s}) \rangle \left\langle \frac{1}{r_1^3} \right\rangle \\ &= \frac{\alpha}{2m_1^2} \cdot \frac{j(j+1) - l(l+1) - s(s+1)}{2} \cdot \frac{2\alpha^3m_1^3}{n^3l(2l+1)(l+1)} \\ &= \frac{\alpha^4m_1}{2n^3} \begin{cases} \frac{1}{(l+1)(2l+1)} & (\text{for } j = l + \frac{1}{2}) \\ -\frac{1}{l(2l+1)} & (\text{for } j = l - \frac{1}{2}) \end{cases} \end{aligned} \quad (24)$$

The transformation to the c.m.s. (in the lowest order) leads to the substitution $m_1 \rightarrow \mu$ in δE . Adding the received value to the spectrum shown in eq. (21) we get the result coinciding with spectrum (18).

From this example we can see that the dependence on the reference frame choice does not affect the final result for the observable quantity. Calculations do not depend on the relation between particle masses, and relativistic recoil effects are included exactly.

3.3 Two spinor charged particles

Here we will very briefly show how to calculate the positronium spectrum using our approach. In this case it is convenient to start from the result for two scalar particles and then add spin-orbital and spin-spin contributions calculated as perturbations. For equal masses in the c.m.s. from Eq. (21) we obtain

$$E_{CM} = 2m - \frac{\alpha^2}{4n^2}m - \frac{\alpha^4}{2n^3}m \left(\frac{1}{2l+1} - \frac{11}{32n} + \varepsilon_{lSJ} \right) + \dots, \quad (25)$$

where ε_{lSJ} is added.

The operator corresponding to the additional term is just the well known operator of the hyperfine structure in the hydrogen atom. And it is known (see *e.g.* [11]), that the average value of the operator for the case of the positronium state gives the ε term for the ortho-positronium spectrum:

$$\begin{aligned} \varepsilon_{l,S=0,J} &= 0, \\ \varepsilon_{l,S=1,J} &= \frac{1}{2l(2l+1)} \begin{cases} \frac{3l+4}{(l+1)(2l+3)} & (\text{for } J = l+1) \\ -\frac{1}{l(l+1)} & (\text{for } J = l) \\ -\frac{3l-1}{l(2l-1)} & (\text{for } J = l-1) \end{cases} \end{aligned} \quad (26)$$

The contribution due to the annihilation channel can be added as a perturbation as well.

4 Conclusions

The suggested approach allows to reduce any two-particle problem with local interactions to a one-particle one. If the particles were different one could choose the type of the one-particle problem which is preferred. We demonstrated that the approach works properly as in classical mechanics as well as in the relativistic quantum mechanics. In the rest RF of one particle we have to describe the motion of the second particle in a static external potential. And there in a relativistic case we do not have any problem with the retardation of interactions and with the different times of our particles, because all coordinates of the first particle were eliminated by construction.

The two-particle problem was treated above without quantum loop effects which are known to appear within the quantum field theory. Here one can proceed in two ways. The first possibility is to choose some basic semi-classical approximation and compute all quantum and relativistic effects as perturbations. Actually this approach has been widely applied to describe atomic spectra with ultimate precision [13]. The other possibility is to exploit the Bethe-Salpeter approach [3]. But in both cases the choice of a reference frame should be done. As we have demonstrated, choosing the rest reference frame of one of the particles might have severe advantages with respect to the traditional c.m.s. version.

The unified approach to scattering and bound states was used. And it leads to a description of a bound state in which the particles have some eigenvalues of their momenta. And it allows to do that simple transformations between different reference frames. We should note that the solution for a wave function of a bound state found in our approach in the first particle

rest RF can not be considered as the wave function of a really moving two-particle system, because the velocity of the system takes imaginary values.

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References

- [1] I. Newton, *Principia* , Book I, Sect. II.
- [2] A.A. Logunov and A.N. Tavkhelidze, Quasi-optical approach in quantum field theory, *Nuovo Cimento*, **29** (1963) 380.
- [3] H.A. Bethe and E.E. Salpeter, A relativistic equation for bound-state problems, *Phys. Rev.* **82** (1951) 309; *Phys. Rev.* **84** (1951) 1232.
- [4] P. Droz-Vincent, Action at a distance and relativistic wave equations for spinless quarks, *Rep. Math. Phys.* **8** (1975) 79.
- [5] A. Komar, Constraint formalism of classical mechanics, Interacting relativistic particles, *Phys. Rev.* **D18** (1978) 1881, 1887.
- [6] I.T. Todorov, Quasipotential equation corresponding to the relativistic eikonal approximation, *Phys. Rev.* **D10** (1971) 2351.
- [7] H. Sazdjian, Relativistic wave equations for the dynamics of two interacting particles, *Phys. Rev.* **D33** (1986) 3401.
- [8] H. W. Crater and P. Van Alstine, *Phys. Rev. D* **36** (1987) 3007.
- [9] E.T. Whittaker, *A Treatise on the Analytical Dynamics of particles and rigid bodies*, the fourth edition, USA, New York, 1944.
- [10] A.B. Arbuzov, On a novel relativistic quasipotential equation for two scalar particles, *Nuovo Cimento A* **107** (1994) 1263.
- [11] H.A. Bethe and E.E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms*, USA, New York, 1977.
- [12] H.A. Bethe, *Intermediate Quantum Mechanics*, W.A. Benjamin, INC. New York — Amsterdam, 1964.
- [13] M.I. Eides et al., *Theory of Light Hydrogenlike Atoms*, *Phys. Rep.* **342** (2001) 61.